Mathematics and the World – The Concept of Experiment[1]

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Abstract

In the paper I discuss Ule's analysis of the relationship between mathematics and (natural) sciences, with emphasis on the context of discovery and the way we come to know about the basic concepts of mathematics and natural sciences. I argue that in such epistemic context, the analogy between mathematics and natural sciences holds thoroughly. I concentrate on just one possible epistemic path – the experiment – and analyze this concept by trying to show how and in which sense the experiments used in the natural sciences are analogous with some of the basic procedures in the mathematical practice.

Keywords: mathematics, natural sciences, epistemology, experiment

Matematika in svet – pojem poskusa – povzetek

V članku obravnavam Uletovo analizo odnosa med matematiko in (naravoslovnimi) znanostmi, pri čemer je poudarek na kontekstu odkrivanja in poti, ki privede do znanja o temeljnih pojmih matematike in naravoslovnih znanosti. Zagovarjam stališče, da v takih epistemskih kontekstih analogija med matematiko in naravoslovnimi znanosti popolnoma drži. Osredotočim se na eno možno epistemološko pot – poskus – in analiziram ta pojem ter skušam pokazati, kako in v kakšnem smislu so poskusi, uporabljeni v naravoslovnih znanostih, analogni nekaterim temeljnim postopkom v matematični praksi.

Ključne besede: matematika, naravoslovne znanosti, epistemologija, poskus

Introduction

Ule, in his article "How can we apply mathematics to the world?" (2002), analyzes in detail the relationship between mathematics and science. He focuses on various aspects of that relationship, and primarily on the problem of the applicability of mathematics as well as on "the transpositions of real objects, their properties and relations onto the new level of their abstract mathematical equivalencies" (*ibid:* 38).

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In the paper[2] I will focus on the mathematics-natural sciences relationship (exclusively) from the epistemological perspective. If we look at this relationship through the prism of epistemology, the two domains turn out to be strongly analogous, the analogy being particularly noticeable in the context of discovery. The aim of this paper is, hence, to present a segment of the analogy between the possible epistemic routes of gaining mathematical knowledge and the truths of (natural) sciences.

When talking about the methodology of the epistemology of science and of mathematics, I support (Kitcher's) pragmatic naturalism,[3] i.e. the view that we ought to look at the history in order to determine the epistemology since "epistemology without history is blind" (Kitcher, 2011: 523).

The underlying idea is that the epistemological route follows the historical one, and that "the epistemological order of mathematics broadly recapitulates the historical order" (Kitcher, 2011: 518). This is not to say that, in order to answer *every* epistemic question, we ought to look at the historical development (in our case, of mathematics and/or natural sciences). There are research areas in epistemology, e.g. those belonging to the context of justification, which are not related to any historical investigation. If, however, our goal is to analyze – given the context of discovery – the way(s) in which we come to grasp the basic concepts and truths in mathematics and the natural sciences, then it is reasonable to investigate the history of these disciplines, i.e. the way in which mathematicians and scientists have, as a matter of fact, come to acquire knowledge through the centuries

In the paper I shall defend the idea that one of the main modes of epistemic access to both mathematical and scientific reality (objects and properties) is the experiment.

The concept of experiment in science and mathematics

Since mathematics is generally taken to be *a priori*, in contrast with the predominantly empirical profile of scientific knowledge, it might seem as if we ought to look for the mathematics-science analogy elsewhere.

When describing the relationship between reality (described by the mathematical apparatus) and the world (having the empirical connotation), Ule aptly points out:

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2 The paper was first presented at the Symposium in honor of Prof. Dr. Andrej Ule (2016). He taught me philosophy when I was an undergraduate mathematics student, and was later supervisor of my MA thesis and PhD dissertation on the philosophy of mathematics. Passionately interested in both mathematics and philosophy, Prof. Ule has always been an inspiring and thought-provoking interlocutor. On this occasion, I would like to express once again my gratitude for all his help throughout the years.

3 I would like to stress at this point that my underlying ontology is, contrary to Kitcher's, platonistic. However, the history-is-the-teacher-of-epistemology motto and the idea that we ought to look at the history of mathematics and (natural) sciences in order to determine the epistemic paths are not tied to any particular ontological theory, so the ontological difference is immaterial.

> I propose that we must distinguish between *reality* and the *world*. The world, on the one hand, is that entity consisting of all causally-connected phenomena and facts, which can be described with the help of the same ontology employing predicative structures of language. Reality, on the other hand, is that "excess" which is not described (well) by our successful theories and which can be described ontologically neither as useful fiction nor by assigning it any standard ontological meaning. Quantum reality, for example, exists in a manner which is formally describable, at least partially in terms of a wave function (a function of state) and of corresponding equations. "Electrons" and other micro-particles form part of this reality and can be individuated as objects only partially and temporarily. Thus, one could say, along with realists, that an electron as a particle (or a wave) does not exist, but that rather a quantum reality exists, which behaves, under suitable conditions, in a manner befitting "electron-ness." The very fact that it is possible to describe reality with the help of mathematics – even in cases when every language and (representational, conceptual) thought falls short – indicates that even mathematics itself is, in some sense, a part of reality, "outside" of the world. (Ule, 1996: 212–213)

Given that mathematics is usually taken to be an *a priori*, armchair activity – it might seem problematic to relate the mathematical domain to any experimental epistemic route. So, what is the idea of such a connection based on?

Before answering this question, let us have a closer look at what experiments amount to. I shall use the entry from the *Stanford Encyclopedia of Philosophy* to provide a mainstream characterization of the role of experiment.

> Experiment plays many roles in science. One of its important roles is to *test theories* and to provide the basis for scientific knowledge. It can also *call for a new theory*, either by showing that an accepted theory is incorrect, or by exhibiting a new phenomenon that is in need of explanation. Experiment can provide hints toward the structure or mathematical form of a theory and it can *provide evidence for the existence of the entities involved in our theories*. Finally, it may also have a life of its own, independent of theory. Scientists may investigate a phenomenon just because it looks interesting. Such experiments may provide evidence for a future theory to explain. /…/ a single experiment may play several of these roles at once. (Franklin, 2012; all emphases mine)

Generally, what we have learned during science classes at school is that experiments ought to be empirical, i.e. concrete. I suppose we all remember the (more or less) simple experiments done during the physics, chemistry or biology classes in high school. These could be the very simple static electricity experiments where we would rub a pen on a woolen sweater and then pick up small pieces of paper; or, in biology, a simple seed germination analysis in order to experience how seeds germinate and what is required. We might also remember experiments that were a bit more complex, such as building an electromagnet or, during chemistry experiments, performing the electrolysis. The underlying idea is that experiments are practical procedures.

Surprisingly, however, if we look at the way experiments have been perceived by *scientists* through history, there is no uniform picture. Furthermore, there is not even general agreement on experiments being *real-world, practical* methods for acquiring knowledge.

Galileo, in his *Two New Sciences* (1638/1914), mentions three kinds of experiments: real, imaginary and thought experiments. Real experiments are those explained in the book, which Galileo did actually perform. Those experiments that Galileo explained in the book, but did not actually perform, even though he could have, are the imaginary experiments. Thought experiments are those that Galileo could not have actually performed for either logical reasons or due to lack of equipment.

It is relevant to mention at this point that there is still no consensus regarding which experiments in his book are real and which are imaginary. Mach, in Chapter II of his *The Science of Mechanics* (1893/1960), talks about "the modern spirit" of Galileo in the sense that "the method he employs to ascertain this law[4] is this. He makes certain assumptions. He does not, however, like Aristotle, rest there, but endeavors to ascertain by trial whether they are correct or not" (Mach, 1893/1960: 130). And while Koyré argues that the inclined-plane experiment in Galileo's writing is totally worthless and generally doubts that Galileo had actually performed many of the experiments described in his *The Two New Sciences,* Settle indicates that the inclined-plane experiment was likely a real one.[5]

Even more interesting is the situation with thought experiments since such experiments, i.e. those that cannot be performed, played a major role in the development of scientific theories in the work of (not just) Galileo, but also Newton, Einstein, and Heisenberg.

After all, scientific experiments are usually defined as orderly procedures (or tests) with, as mentioned above, certain goals, but this certainly does not exclude the possibility of them being non-empirical. Nevertheless, what happens in experimental science might seem at first sight remote from the standard mathematical practice; if anything, given the fact that mathematical objects are abstract, i.e. they are not spatiotemporally located.

When talking about experiments in natural sciences, the main distinction is the one between confirmatory (or demonstrative) and exploratory (non-demonstrative) experiments. The former are those in which we test theories, while the latter are those in which the experimentation is not guided by hypotheses, but it is rather about searching. The goal here is to show that, no matter which of the two main sub-species of the experiment we prefer to concentrate on – either the confirmatory or the exploratory (non-demonstrative) one –, the analogy with the mathematical case holds throughout.

4 Mach refers to the law of falling bodies.

5 For further details, see MacLachlan (1973).

Confirmatory (demonstrative) experiments

If we look for the analogy with mathematics in the case of the confirmatory (or demonstrative) use of experiments, many mathematical proofs can be interpreted as confirmatory experiments. Examples are legion.

Let us mention the problem of doubling the cube, one of the three classical problems in ancient Greek mathematics.[6] The problem of doubling the cube is also known as the problem of duplicating the cube or as the Delian problem. Even though it is almost certain that ancient Greek mathematicians were convinced that the ruler and compass construction was impossible, the proof of the impossibility was only discovered in the nineteenth century by Wantzel (Pierre Wantzel, French mathematician). He published the proof in 1837, while Gauss (Carl Friedrich Gauss, German mathematician) thought the problem had no solution, but provided no proof. It is no surprise that the Greeks did not find the proof, given that it requires knowledge of mathematics beyond anything they knew at the time. Some of the ancient Greek mathematicians that took most interest in the problem were Hippocrates, Archimedes, and Archytas. Apart from them, the Egyptians and Indians were also aware of the problem.

Another – this time "negative" – mathematical case would be that of Saccheri, whose aim was to prove the dependence of the 5th Euclidean postulate (hypothesis). While planning to prove the dependence of the 5th postulate on the other four, Saccheri presupposed it being independent. He hence presupposed the first four postulates being true while the 5th one false. Saccheri's aim was to get the contradiction by using the *reductio ad absurdum*. Due to the actual independence of the 5th postulate, his goal resulted in the negative. He was not able to prove what he was aiming at, and at the same time was not aware of the discovery of a new, non-Euclidean, geometry.

Exploratory (non-demonstrative) experiments

Other mathematical results and proofs are analogous to the exploratory, non-demonstrative experiments. In such experiments, the experimentation is not guided by hypotheses.

A nice example in mathematics is the problem of determining prime numbers and their properties. Prime numbers have been of great interest to mathematicians, and have been studied thoroughly since the Pythagoreans. Euclid, in Book IX of his *Elements*, provided the proof that there are infinitely many prime numbers. The question that still remained was: which ones of the (infinite natural) numbers are prime? Even though the ancient Greek mathematician Eratosthenes found an algorithm for determining the primes – the sieve of Eratosthenes, the sieve was most efficient for finding the smaller primes, but not so much for bigger numbers.

6 The other two are that of squaring the circle and trisecting the angle.

In the seventeenth century, Fermat (Pierre Fermat, French mathematician) proved several theorems concerning the primes. One of them is part of the two-thousand year old hypothesis that a number *n* is prime if the number 2n-2 is divisible by *n*. Other famous mathematicians who had great impact on prime number theory were Euler, Legendre and Gauss. Apparently, Gauss managed to calculate all the primes up to about three million. New claims concerning the primes and their density were proven by Chebyshev and Riemann in the nineteenth century. There are still many open questions regarding the primes (some of them hundreds of years old), such as the conjecture that there are infinitely many pairs of primes only two apart (e.g. 3 and 5, 5 and 7, 11 and 13, 17 and 19, 41 and 43 etc.).

Another equally interesting example in the history of mathematics is the problem of trisecting an arbitrary angle. The attempts to solve the problem can be seen as an exploratory experiment that has been going on for centuries.

The long-lasting process of finding the prime numbers can be compared with the 2,500-year long process in chemistry to determine what things are made of. Even though Democritus' idea was that all matter was made of tiny particles – *atoms*, the predominant view had been for centuries that of Aristotle (who accepted Empedocles' view) that everything which exists was made from just four "elements." And it was only in the eighteenth century that Scheele (Karl Scheele, Swedish chemist) and Priestley (Joseph Priestley, English chemist) discovered oxygen, while Lavoisier (Antoine Laurent Lavoisier, French chemist) who was exploring the true nature of burning, compiled the list of the twenty-eight elements known at the time. With the help of electricity, Davy (Humphrey Davy, English chemist) discovered sodium, potassium, calcium, and magnesium. Later, in 1828, Wohler (Friedrich Wohler, German chemist) produced urea in his laboratory, and additional elements were discovered. Then in 1898, Marie and Pierre Curie (the Polish-born Marie and her French husband) discovered radium. In the following years (until 2010) more than twenty new elements were discovered.

Even though at first sight the analogy between the discovery of prime numbers and that of elements seems plausible, at the end of the day we still might find the proposed analogy between the experiments in (the natural) sciences and those in mathematics unsatisfactory. If nothing else, while in scientific experiments it is possible to directly interfere with objects, this is not possible in the case of mathematical experiments, since abstract objects are involved.

Therefore, if we take a crude example, such as feeding rats with crops grown on animal pasture and observe the effects of plant estrogen on animal reproduction, it is not clear what would such direct manipulation of objects be in the case of mathematical experiments. It looks as if this element is missing from the analogy to make it complete. Moreover, it is kind of implicit that experiments are about manipulations with spatiotemporally located objects, not abstract ones.

On the other hand, if experiments are allowed to be imaginary, as we have just seen to be the case, then it is clear that the concreteness of the objects of manipulations is not a required condition, nor is it a tacit one. In fact, it is difficult to see in what way we literally manipulate objects in imaginary experiments. I would say that we do not. Therefore, we can talk about experiments without presupposing any kind of direct manipulation of concrete objects.

The non-concrete objects which we "manipulate" during imaginary experiments are related to their spatiotemporal counterparts in a way that is analogous to the way in which representations of abstract objects – the subject of manipulations in mathematical experiments – are related to the abstract (mathematical) objects, i.e. their abstract counterparts. In the case of trisecting an arbitrary angle, we do manipulate the representation of an abstract geometrical entity.

All these examples suggest a strong analogy between the experiment in science and some of the central procedures in mathematics.

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