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Introduction to the Five Limit Intervals Harmony

The main structural difference between any equal tempered scale and just intonation system lies in dimensions. Equal temperament is linear or one-dimensional systems, while just intonation can have as many dimensions as we choose it to have.

Just intonation based on 2-limit intervals is one dimensional, 3-limit intervals forms a two-dimensional intonation system, 5-limit intervals forms a 3D system, 7-limit forms 4D system, 11-limit forms 5D system, etc. Compared to each other, both systems have their advantages and disadvantages, but in terms of harmony, the most important difference between these two systems, besides the sound of the intonation itself, is the different way of handling the chord inversions. Equal temperament provides all possible chord inversions, which are restrained only by the stylistic rules of the harmony, while within the just intonation system it is not so. Since just intonation is an n-dimensional system, in order to work within one, it is necessary to choose the limit of the intervals and that very choice defines the restrictions on handling the chord inversions. Choosing the prime factor of the intervals, arbitrary sets the interval borders and raises the question of the interval tolerance. Correlation between interval limits and chord inversions defines the main subject of this paper which partially makes an introduction to my book *Five Limit Intervals – Theory & Praxis*.[1]

A C dominant seventh chord with tension nine or C7(9) consisting of tones C–E–G–B-flat–D is one example where a higher limit interval decreases the number of inversions. C7(9) consists of 1, 4/5, 2/3, 5/9 and 4/9 within 5-limit intervals, while within 7-limit intervals the dominant seventh would be the one of 4/7. The dominant seventh of 5/9 generates inversions with the root on G and E, which is the G minor chord with tension eleven and thirteen or Gm(11,13) and E half-diminished seventh chord with tension flat thirteen or Em7-5(b13). The dominant seventh of 4/7 cannot be used within Gm(11,13) for the minor third between G and Bb is a minor third of 6/7 and not a minor

third of 5/6. The interval of the dominant seventh can be determinate by the chord construction as well.

In order to have C7(#9) with a major triad in the upper structure, the only solution is to use the dominant seventh of 5/9 because only the tension sharp nine of 5/12 makes the major third of 4/5 with the chord a perfect fifth, and the only dominant seventh that makes a perfect fifth with 5/12 is the dominant seventh of 5/9, so the chord will generate all inversions with the following construction within 5-limit intervals: 1, 4/5, 2/3, 5/9 and 5/12.

In order to have C7(b9,13) with the major triad in the upper structure, only the tension flat nine of 12/25 makes minor third of 5/6 with the major third of 4/5 allowing these two tones to form A major triad with tension thirteen of 3/10. In order to keep all other inversions of this chord possible (A minor triad and B-flat half diminished triad), only dominant seventh that makes minor third of 5/6 with tension flat nine of 12/25 is the dominant seventh of 72/125. The chord will maintain all possible inversions, just like in equal temperament and the following chord construction will be again within 5-limit intervals: 1, 72/125, 2/5, 3/10, and 6/25.

The interval of the major or dominant seventh and the major or minor second can be dictated by the chord progression as well. Simple chord progression, like the subdominant–dominant–tonic in the key of C major, could have these voices: (G–E–C–f–a) followed by (A–E–b–f–g) resolving in (G–D–b–e–c) or written with esymbols Fmaj(9)/A, G7(9,13) and Cmaj(9). In order to maintain all inversions with major or minor triads in the upper structures and to keep the same tone "f" in the subdominant and in dominant chord, the only solution is to use the dominant seventh of 9/16 because only F of 3/4 and G of 2/3 makes a perfect fifth with the root of 1.

From examples like this it is possible to conclude that only 5-limit intervals can maintain free usage of chord inversions in the same way as equal temperament does. Establishing the area of zero tolerance for interval alteration between 5/6 and 3/5 generates an area of interval tolerance where every smaller or bigger interval becomes the subject of alteration and must be interpreted as a minor or major second, which is a major or dominant seventh.

Diminished and augmented chords expand the area of tolerance to the area between 3/4 and 2/3 where every interval is interpreted as an augmented fourth or diminished fifth. Construction of the major chord with the diminished fifth or augmented fourth that makes a major third of 4/5 with the dominant seventh of the chord have an immense number of possibilities. Four different possibilities of dominant sevenths (72/125, 128/225, 9/16 and

5/9) in an augmented or diminished chord are given in Example 1 along with four different major seconds (9/10, 8/9, 225/256 and 125/144) directly generated by the choice of dominant seventh. Since all intervals are 5-limit intervals and the border of the intervals is respected as mentioned before, all inversions are possible. That enables the construction of a chord consisting of two different dominant sevenths.
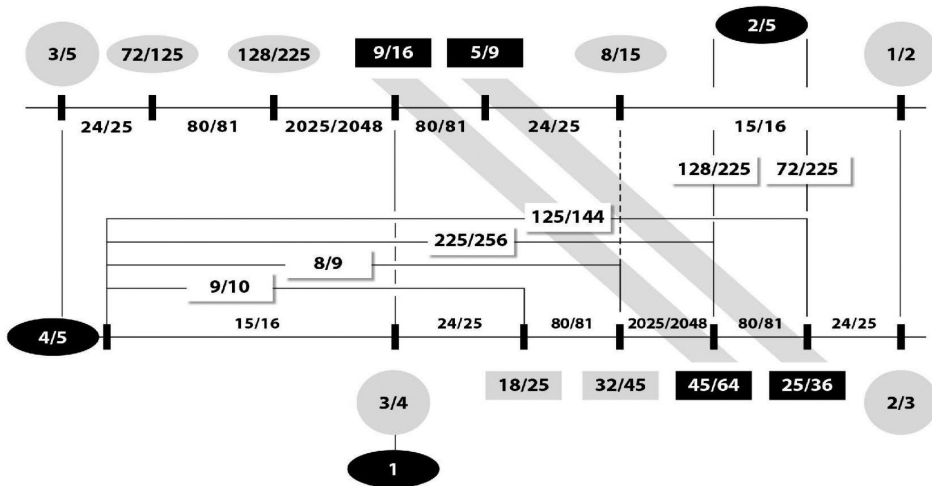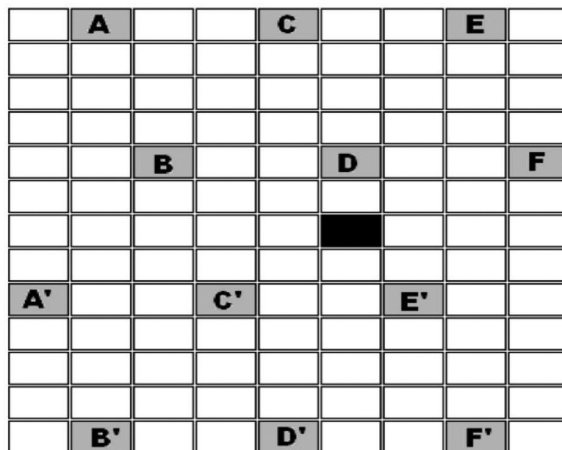

Example 1: Four different dominant sevenths and major seconds and two different augmented fourths and diminished fifths

A dominant augmented chord with intervals 1, 4/5, 18/25, and 72/125 generates a major second of 9/10 between the major third and augmented fourth. Since all the inversions are possible, the augmented fourth of 18/25 generates a dominant seventh of 5/9 with a major third raised above for an octave. This is one of the many examples where the dominant chord can have two different dominant sevenths, that is, 72/125 and 5/9. Section A (Example 2) shows symbols for microtonal sharps and flats, 12 different dominant sevenths within 5-limit intervals and their linear position within the interval of 204 cents between major sixth of the 3/5 and major seventh of the 8/15. Respecting the interval border and placing alterations only within the area of interval tolerance makes all possible chord inversions available and therefore each of them provides an additional dominant seventh interval in the augmented or diminished chord construction mentioned above. Different dominant sevenths have different sound and different function as well.
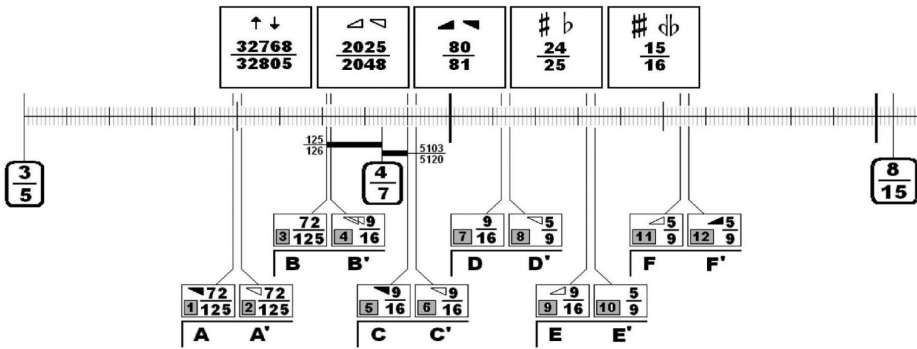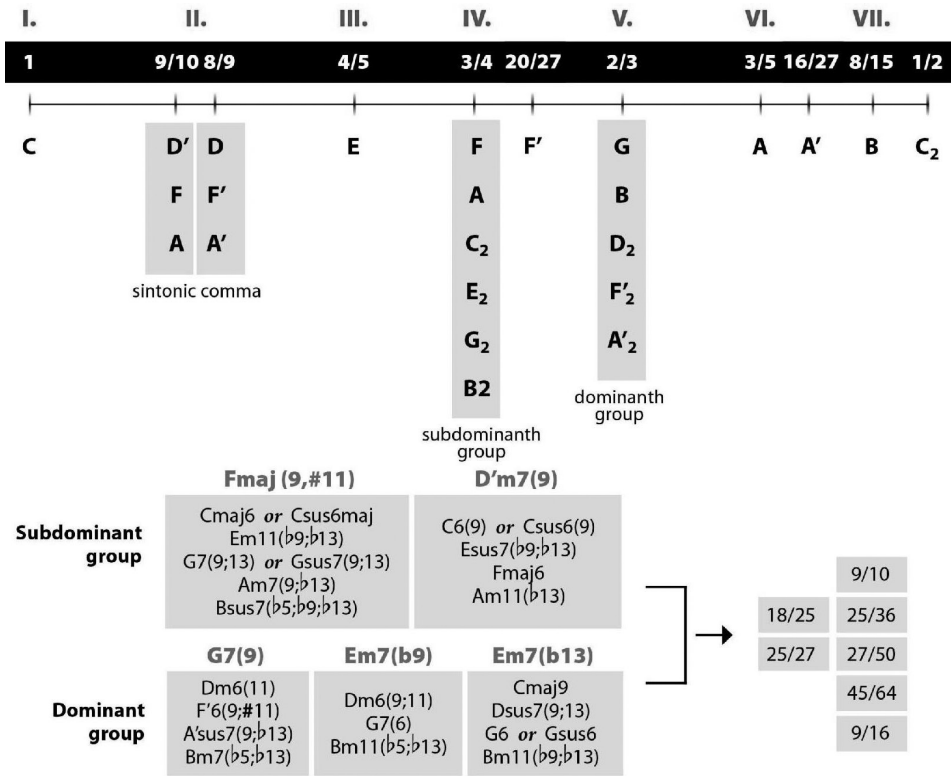
Example 2: Twelve different dominant sevenths marked with letters and five different accidentals

Example 3 presents all 12 dominant sevenths within 5-limit lattice. Dominant sevenths, marked as A, C, E, B', D' and F', enable modulations and chord progressions to a remote area of the 5-limit lattice in relation to the dominate sevenths, marked as D or E', for example. A 5-limit interval harmony can be organized into a three major chord group structures in order to enable all possible chord inversions. The first chord group structure makes a 5-limit interval harmony of the 1st order and it consists of a subdominant and dominant chord group. Example 4 shows all possible inversions in relation to which tone of the chord group is considered to be the root. A harmony of the 1st order covers the area of diatonic changes within the range of one key.


Example 3: Position of twelve different dominant sevenths within 5 limit lattice. Black field marked as 1/1

I.	II.	III.	IV.	V.	VI.	VII.
1	9/10 8/9	4/5	3/4 20/27	2/3	3/5 16/27 8/15	1/2

C	D' D	E	F F'	G	A A' B C$_2$
	F F'		A	B	
	A A'		C$_2$	D$_2$	
			E$_2$	F'$_2$	
			G$_2$	A'$_2$	
			B2		

sintonic comma

subdominanth group

dominanth group

	Fmaj (9,#11)	**D'm7(9)**
Subdominant group	Cmaj6 *or* Csus6maj Em11(b9;b13) G7(9;13) *or* Gsus7(9;13) Am7(9;b13) Bsus7(b5;b9;b13)	C6(9) *or* Csus6(9) Esus7(b9;b13) Fmaj6 Am11(b13)

	G7(9)	**Em7(b9)**	**Em7(b13)**
Dominant group	Dm6(11) F'6(9;#11) A'sus7(9;b13) Bm7(b5;b13)	Dm6(9;11) G7(6) Bm11(b5;b13)	Cmaj9 Dsus7(9;13) G6 *or* Gsus6 Bm11(b9;b13)

	9/10
18/25	25/36
25/27	27/50
	45/64
	9/16

Example 4: Tonality defined with 7 degrees, 10 tones and all possible inversions of dominant and subdominant chord group structures

Within 5 limit intervals tonality is defined with 10 different tones or pitches. These 10 tones are: 1, 9/10, 8/9, 4/5, 3/4, 20/27, 2/3, 3/5, 16/27, and 8/15. Example 5 shows all dominant and subdominant chord inversions within the 5-limit lattice.

The 5-limit interval harmony of the 2nd order covers construction of altered chords and modulation. There are two basic altered chord groups as shown in Example 6 with all possible chord inversions in relation to the tone of the chord group, which is considered to be the root and written with sharps and flats as shown in Example 2 above.
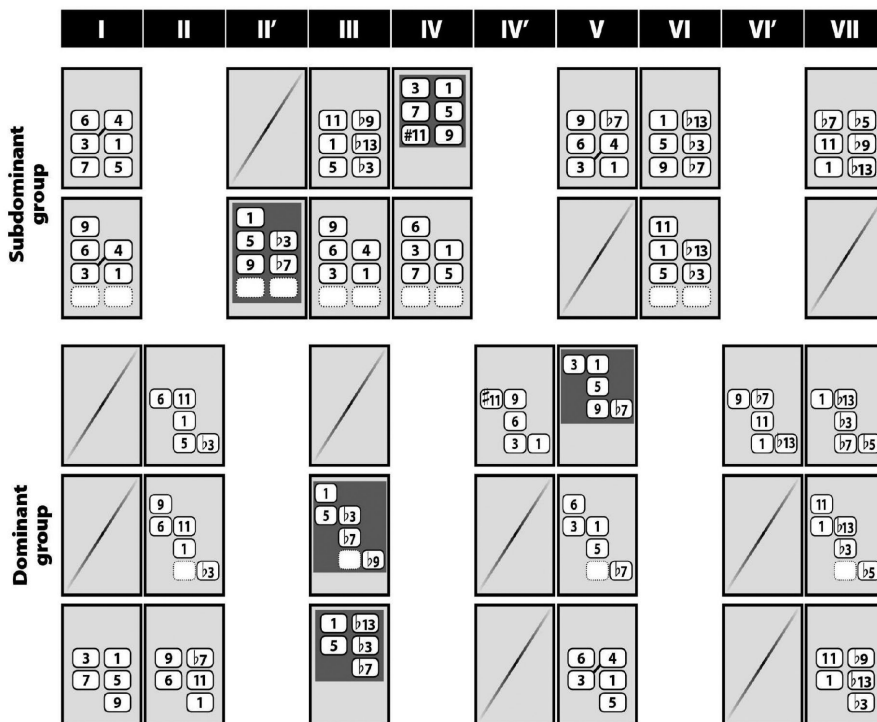
Example 5: All inversions in five limit intervals harmony of the 1st order within 5 limit lattice

SUBDOMINANT CHORD GROUP STRUCTURE

P	X
Q	Y
R	Z

X (1,3,5,7,9,⌐#11)
Y (1,3,5,7,13)
Z (1,3,4,⌐b7,⌐9,13)
P (1,b3,5,b7,9,b13)
Q (1,b3,5,⌐b9,11,b13)
R (1,4,⌐b7,⌐b9,⌐b5,13)

DOMINANT CHORD GROUP STRUCTURE

P	X
	Y
R	Z

X (1,3,5,b7,9)
Y 1,b3,5,11,13)
Z 1,3,6,⌐9,#11)
P (1,3,b7,b5,b13)
R (1,4,⌐b7,⌐9,b13)

ALTERED CHORD GROUP STRUCTURE

P	X
Q	Y
R	Z

X (1,3,5,b7,#9,⌐#11)
Y 1,3,5,⌐b9,#9,13)
Z 1,3,⌐b7,⌐b9,#11,13)
P 1,b3,b5,⌐7,⌐9,b13)
Q 1,b3,6,⌐7,11,b13)
R 1,⌐b5,6,⌐9,11,b13)

ALTERED CHORD GROUP STRUCTURE

Z	
P	X
Q	Y
	R

X (1,3,5,b7,#9,13)
Y (1,3,5,6,⌐b9, #11)
Z (1,b3,5,b7,⌐#11,b9)
P (1,b3,b5,⌐7,11,b13)
Q 1,b3,6,⌐9,11,b13)
R (1,⌐b5,6,⌐7,⌐9,11)

FREE CHORD GROUP STRUCTURE

| P | X |
| | Y |

X (1,3,5,b7,⌐b7,##11)
etc. ...

Example 6: 5-limit interval harmony of the 1st, the 2nd and the 3rd order with all inversions

If we take, for example, tone X from the second altered chord group to be considered as the root, the chord construction would be made of these intervals: 1, 4/5, 2/3, 5/9, 5/12, and 3/10. Within the 5-limit interval harmony of the 1st and 2nd order, every possible chord has all possible inversions. The 5-limit interval harmony of the 3rd order or free chord group structure can generate chord structures which cannot be used for all possible inversions and that is why the harmony of the 3rd order is to be considered with special care.

Certain diminished or augmented chords with dominant sevenths, as shown in Figure 2, can be a part of the 2nd order harmony or a part of the 3rd order harmony. Chord constructions that respect interval tolerance and interval borders have all possible inversions at their disposal, even if they belong to the 5-limit harmony of the 3rd order.

A dominant seventh chord with tension b13 of 16/25 on the other hand, belongs to the 3rd order harmony but it cannot be used for inversions any longer since intervals between 16/25 and 1/2 is not major third of 4/5. A minor chord with a major seventh (1, 5/6, 2/3 and 8/15) is another example of the 3rd order harmony where inversions have been restricted since 8/15 and 5/12 do not form a major third of 4/5.

Another subject of the 5-limit intervals harmony that needs to be considered with special care is syntonic shift. The interval of 80/81 or syntonic comma can occur in many different cases of chord progression within all three orders of the 5-limit intervals harmony. Three basic types of syntonic shifts are: direct syntonic shift (DSS), indirect syntonic shift (ISS) and simultaneous syntonic shift (SSS).

Every chord progression consisting of more than one chord group is a necessary subject of syntonic shift.

Example 7 is part of the composition *Strong Man*, mm. 17–32. The piano is tuned according to the following ratios:

F = 1

F# = 9/10

G = 8/9

G# = 5/6

A = 4/5

A# = 3/4

B = 20/27

C = 2/3

C# = 16/25

#D = 3/5

D# = 5/9

E = 8/15

Four kinds of noteheads are used in the analysis. Normal noteheads are used for the notes from the actual piano score. Diamond noteheads are used for the rest of the notes from the possible chord group. Slashed noteheads are used for the notes that could be part of the maximal chord group structure according to piano tuning, regardless of tonality group. Cross noteheads are used for the tones that are not considered to be a part of the chord group, but rather as an approach to the next note. Chord symbols are given according to normal noteheads. Four different types of sharps and flats, as introduced in Example A, are used only with notes and alphabet chord symbols, but not with the numbers within the chord symbols. The 5-limit lattice diagram placed above the chord symbol is given according to the chord group written with normal and diamond noteheads. Black fields in each diagram represent the root of the chord group according to chord symbol.

Example 7: Šćekić's Strong Man, *mm. 17–32 with harmony analysis*

The first three bars of Example 7 are written within the dominant group. According to Example 5, the first chord group is an inversion of the dominant minor parallel, the one with the tension (b13) and with the root on the 1st degree. The second chord group is an inversion of the same chord group with the root on the 5th degree with the omitted major third. The next chord group is the same inversion but with omitted fourth. If we add slashed noteheads (syn. B-flat and E-flat) into a chord group, the structure of the chord group will become altered. That is the possibility that we have, according to the piano tuning. The next chord group, the one in m. 19, is the same chord group as the one in m. 17. The chord group in m. 20 belongs to the subdominant group, and since the transition from one group to another necessarily produces syntonic shift, this is the place where it should occur. In this case it consists of a syntonic comma lower, or regarding inversions, higher intervals between G (8/9) and B-flat (3/4) or G and D (3/5). Both cases are examples of indirect syntonic shift. Both shifts are in different voices and the one from G to D is delayed by the use of the note C in between. That is why it is not so easy to hear the micro-modulation within ISS. If a syntonic shift would appear in the same voice it would be easier to hear it, but the case would still remain within ISS. Transitions from m. 23 to m. 24, 26 to 27, and 30 to 31 produce ISS in the same voice, that is, the syntonic comma lower minor third between G (8/9) and B-flat (3/4). ISS from m. 23 to m. 24 is delayed by the use of note A in between.

It is impossible to achieve harmonic progression from one chord group to another without indirect syntonic shift, even within the same key.

Example 8 is part of the composition *23.10*. This is the case of direct syntonic shift (DSS) where the tone from one group is resolved to a tone on the "same" degree but from another group and of course, in the same voice. In Example 8, that would be the tones G (8/9) and Bb (20/27) from the inversion of the dominant minor parallel (the one with the tension b9 and with the root on the 2nd degree) resolving to the tones G (9/10) and Bb (3/4) from the subdominant minor parallel.

Examples 9, 10, and 11 are parts of the composition *Autumn Fantasy of Martin the Mouse*. In Example 9, DSS is achieved through alternation of 8/9 and 9/10 in the left hand.

Example 8: Šćekić's 23.10*, mm. 263–66 with harmony analysis*

Example 9: Šćekić's Autumn Fantasy of Martin the Mouse*, with harmony analysis*

According to Example 6, the chords from Example 10 belong to the free chord group structure because there are seven tones in the chord group. Since all of the tones are played simultaneously (Ped.), this chord group produces the third type of the syntonic shift, that is, simultaneous syntonic shift (SSS).

Example 11 belongs to the 3rd order harmony as well. Chord group C7 has tones from both groups, dominant and subdominant. The free chord group from the last two bars in Example 9 consists of the root, major third, perfect fifth, and dominant seventh of the 5/9 but also of the syntonic comma lower perfect fifth and syntonic comma lower dominant seventh of 9/16. Wirh regards to the central tone F, that would be the following tones: 9/10 and 3/4 from the subdominant group and 8/9 and 20/27 from the dominant group. This is again the third type of syntonic shift since the interval of syntonic comma is achieved simultaneously (SSS).

Example 10: Šćekić's Autumn Fantasy of Martin the Mouse, *mm. 118–119 with harmony analysis*

Example 11: Šćekić's Autumn Fantasy of Martin the Mouse, *mm. 128–130 with harmony analysis*

The sounds of the DSS and SSS are so characteristic that they can be used only in order to make an accent on the shift itself. An example of that kind of use of syntonic shift can be found in *E majors Study #1*. The study is written for four pianos or keyboards where each is tuned in a different way. This provides a wide range of pitches within 5-limit intervals. Example 12 shows a palette of the pitches that can be achieved with four pianos or keyboards

with sharps and flats as introduced in example A. There are five different pitches for tone E in *E majors Study #1*.

Example 12: A 5-limit lattice with a choice of 42 different tones per octave for
E majors Study #1 *for four pianos in 5-limit just intonation*

The first pitch for tone E is the one on the top of the second row (from left to right). The second pitch for tone E in the third row is a syntonic comma higher. The third pitch for tone E in the fifth row is a diaschisma higher, an interval of 2025/2048. The fourth pitch for tone E in the sixth row is a syntonic comma higher, and the last pitch for tone E in the seventh row is again a syntonic comma higher. This range of pitches provides enough space for exploring all cases of syntonic shift within the five limit interval harmonies of the 1st, the 2nd, and the 3rd order. The main subject of *E majors Study* is to explore the shift of one tone within the chord and the pitch shifting of the whole chord or tonality, achieved gradually through the chord progression and instantly through the direct syntonic shift.[2]

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2 A video clip of *E majors Study #1* with sound and animation of chord progression within a 5-limit lattice is available at: https://vimeo.com/55660995.